IDEAL
VIII • IX • X [SSC]

DATE : 20/05/2012 | **IDEAL TEST SERIES** | **MARKS : 30**
STD : X | **SUBJECT: GEOMETRY (SOL)** | **TIME :1 HR**

Ans.1.Solve any two **[2 M]**

(1) $\frac{A_1}{A_2} = \frac{b_1 \times h_1}{b_2 \times h_2}$ (Triangle with diff heights & diff bases) ($\frac{1}{2}$ M)

$\therefore \frac{4}{5} = \frac{3}{2} \times \frac{h_1}{h_2}$

$\therefore \frac{h_1}{h_2} = \frac{4}{5} \times \frac{2}{3}$

$\therefore \frac{h_1}{h_2} = \frac{8}{15}$ (½ M)

(2) Similar triangles are congruent triangles .It is a false statement.

(3) By Geometric Mean definition

$b^2 = a \times c$ (½ M)

$\therefore (6)^2 = a \times c$

$\therefore 36 = a \times c$

$\therefore$ (1, 36) (36,1) (2,18) (18, 2) (4, 9) (9, 4) (3, 12) (12, 3) (½ M)

Ans.2. Solve any three questions **[6 M]**

(1) Let A_1 andA_2 be the areas of larger triangle and Smaller triangle with equal heights and corresponding bases b_1 and b_2 respectively. (½ M)

$\therefore \frac{A_1}{A_2} = \frac{b_1}{b_2}$ (Triangles with equal heights) (½ M)

$\therefore \frac{3}{2} = \frac{18}{x}$ (½ M)

$\therefore 3 \times x = 18 \times 2$

$\therefore x = \frac{18 \times 2}{3}$

$\therefore x = 12$

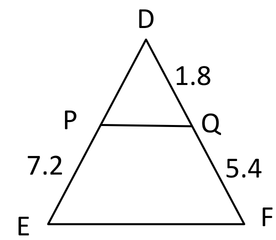
$\therefore$ base of the smaller triangle is 12 cm. (½ M)

(2) Δ ABC ~ Δ PQR (given)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ (Areas of similar triangles theorem) (1 M)

$\therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \left(\frac{4}{6}\right)^2 = \frac{16}{36} = \frac{4}{9}$ (1 M)

(3) In Δ DEF



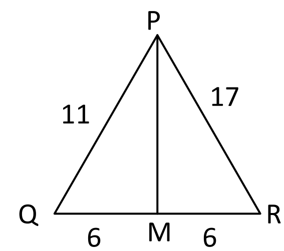
Seg PQ || Side EF (given)

$\therefore \frac{DP}{PE} = \frac{DQ}{QF}$ (Basic proportionality theorem) (1 M)

$\therefore \frac{DP}{7.2} = \frac{1.8}{5.4}$ Substitute the values

DP = $\frac{1.8 \times 7.2}{5.4}$ = 2.4 units (1 M)

(4) QR = 12



$\therefore$ QM = $\frac{1}{2}$ QR (M is a mid point of QR)

= $\frac{1}{2} \times 12$

= 6

In Δ PQR

PM is a median

$\therefore PQ^2 + PR^2 = 2PM^2 + 2QM^2$ (Apollonius principle) (1 M)

$\therefore (11)^2 + (17)^2 = 2PM^2 + 2(6)^2$

$121 + 289 = 2PM^2 + 72$

$410 - 75 = 2PM^2$

$\therefore 2PM^2 = 338$

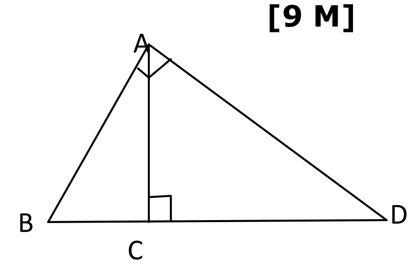
$PM^2 = 169$

Taking Sq root

PM = $\sqrt{169}$ = 13 Units (1 M)

Ans.3. **Solve any three** **[9 M]**

(1) Δ BCA ~ Δ BAD (Right angled a Similarity theorem)



$\therefore \frac{BC}{BA} = \frac{BA}{BD}$ (Corresponding Sides are in proportion)

$BA^2 = BC \times BD$ (1 M)

Δ ACD ~ Δ BAD (Right angled Δ Similarity Theorem)

$\therefore \frac{CD}{AD} = \frac{AD}{BD}$ (Corresponding Sides are in proportion)

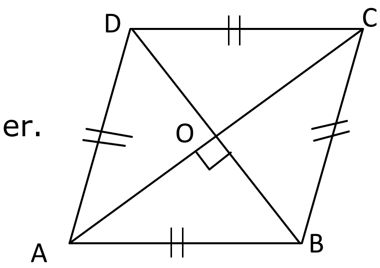
$\therefore AD^2 = BD \times CD$ (1 M)

Δ BCA ~ Δ ACD (Right angle Δ Similarity theorem)

$\therefore \frac{BC}{AC} = \frac{AC}{CD}$ (Corresponding Sides are in proportion)

$\therefore AC^2 = BC \times CD$ (1 M)

(2) Sides AB = 10 cm
Diag AC = 16 cm
In a rhombus



Diagonals are perpendicular bisector of each other.

$\therefore$ AO = $\frac{1}{2} \times$ AC = $\frac{1}{2} \times 16 = 8$ cm (1 M)

In Δ AOB m$\angle$ O = 90°

$AB^2 = AO^2 + BO^2$ (pythagoras theorem)

$\therefore (10)^2 = (8)^2 + BO^2$

$\therefore$ BO = 6 cm (1 M)

BD = $2 \times$ BO = $2 \times 6 = 12$ cm

Length of other diagonal is 12 cm. (1 M)

(3) (i) **Δ** ABC ~ **Δ** DEF (given)

(ii) $\therefore \frac{A(\Delta ABC)}{A(\Delta DEF)} = \frac{BC^2}{EF^2}$ (Areas of Similar triangles theorem) (1 M)

(iii) $\therefore \frac{9}{16} = \frac{BC^2}{EF^2}$

Taking Sq root

$\frac{3}{4} = \frac{BC}{EF}$ (1 M)

$\therefore \frac{3}{4} = \frac{2.1}{EF}$

$3 \times EF = 4 \times 2.1$

$EF = \frac{4 \times 2.1}{3}$

$EF = 4 \times 0.7 = 2.8$

$\therefore$ EF = 2.8 cm (1 M)

(4) In **Δ** ABC Ray BD is the bisector of $\angle$ ABC

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$ (angle bisector property of a triangle) (1 M)

$\therefore \frac{25}{36} = \frac{12.5}{DC}$

$\therefore DC = \frac{36 \times 12.5}{25} = 18$ Unit (1 M)

AC = AD + DC = 12.5 + 18 = 30.5

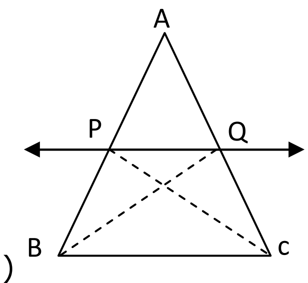
Perimeter of **Δ** ABC = AB + BC+ AC

= 25 + 36 + 30.5

= 91.5 Unit (1 M)

Ans.4. **Solve any two** **[8 M]**

(1) Statement : If a line parallel to a side of a triangle intersect the other sides in two distinct points, then the line divides those sides in proportion (equal ratio)



Given In **Δ** ABC

Line PQ || Side BC

Intersect AB and AC at P and Q respectively

<u>To prove</u> : $\frac{AP}{PB} = \frac{AQ}{QC}$

Contruction Draw Seg BQ and Seg CP (1 M)

	Proof Statement	Reason
(1)	**Δ** APQ and **Δ** BPQ both have common Vertex Q and their bases lie on Same line AB $\therefore \frac{A(\Delta APQ)}{A(\Delta BPQ)} = \frac{AP}{PB}$	(Triangles with equal heights) (1 M)
(2)	**Δ** APQ and **Δ** CPQ both have common Vertex P and their bases lie on Some Line AC $\therefore \frac{A(\Delta APQ)}{A(\Delta CPQ)} = \frac{AQ}{QC}$	(Triangles with equal heights) (1 M)
(3)	Δ BPQ and Δ CPQ both have Same base PQ and lie between two parallel lines PQ and BC ∴ Their heights are equal ∴A(Δ BPQ) = A (Δ CPQ)	(Triangles with same base and equal height)
(4)	$\therefore \frac{A(\Delta APQ)}{A(\Delta BPQ)} = \frac{A(\Delta APQ)}{A(\Delta CPQ)}$	(from 1, 2, 3)
(5)	$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$	(from 1, 2, 4)
Hence Basic proportionately theorem is proved. (1 M)		

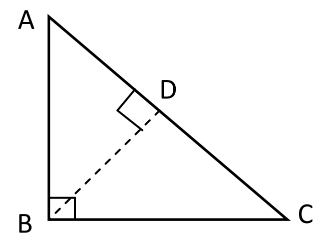
(2) Statement : In a right angled triangle square of the hypotenuse is equal to the sum of the Square of other two Sides.

<u>Given</u>

In Δ ABC m$\angle$ B = 90°

<u>To Prove</u>

$AC^2 = AB^2 + BC^2$



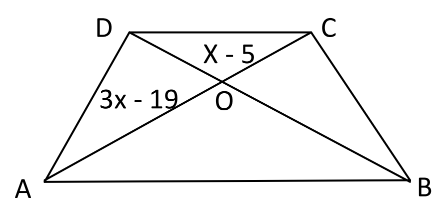
Construction : Draw BD $\perp$ Side AC (1 M)

Proof	Statement	Reasons
(1)	Δ ADB ~ Δ ABC	Right angled Δ Similarity theorem
(2)	$\therefore \frac{AD}{AB} = \frac{AB}{AC}$	(Corresponding Sides are in Proportion)
(3)	$\therefore AB^2 = AC \times AD$	from 2 (1 M)
(4)	Δ BDC ~ Δ ABC	(Right angled Δ Similarity theorem)
(5)	$\therefore \frac{DC}{BC} = \frac{BC}{AC}$	(Corresponding Sides are in proportion)
(6)	$\therefore BC^2 = AC \times DC$	from 5 (1 M)
	Adding 3 and 6	
(7)	$AB^2 + BC^2 = (AC \times AD) + (AC \times DC)$	
(8)	$\therefore AB^2 + BC^2 = AC\ (AD+DC)$	
(9)	$AB^2 + BC^2 = AC \times AC$	
(10)	$AB^2 + BC^2 = AC^2$	(1 M)
	Hence Pythagoras theorem proved	

(3) AO = (3x - 9)
OC = (x – 5)

OD = 3

OB = (x – 3)



Seg AB || Seg CD (given)

(1) $\therefore \angle$ CAB $\equiv \angle$ ACD converse of attenuate angle test

(2) i.e $\angle$ OAB $\equiv \angle$ OCD $\begin{pmatrix} C- & O- & A \\ A- & O- & C \end{pmatrix}$ (1 M)

In Δ AOB and Δ COD

(3)	$\angle$ OAB $\cong \angle$ OCD	from 2
(4)	$\angle$ AOB $\cong \angle$ COD	Vertically opposite angle
(5)	$\therefore \Delta$ AOB ~ Δ COD	(AA test of Similarity)

(6) $\therefore \frac{AO}{CO} = \frac{OB}{OD}$ (Corresponding Sides are in proportion)

$\therefore \frac{3x-19}{x-5} = \frac{x-3}{3}$

$3(3x - 19) = (x - 3)(x - 5)$ (1 M)

$9x - 57 = x^2 - 8x + 15$

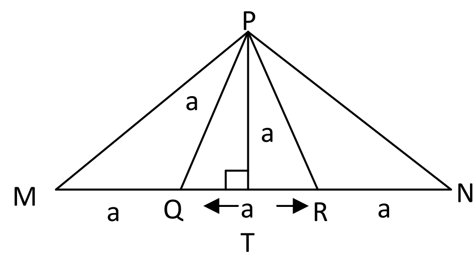
$\therefore x^2 - 17x + 72 = 0$

$\therefore (x - 8)(x - 9) = 0$

$\therefore x = 8$ OR $x = 9$ (1 M)

Ans.5. **Solve any one** **[5 M]**

(1)



In **Δ** PMR

PQ is a Median

$\therefore PM^2 + PR^2 = 2PQ^2 + 2QR^2$ (Apollonius principle)

$\therefore PM^2 + a^2 = 2a^2 + 2a^2$ (1 M)

$\therefore PM^2 = 4a^2 - a^2$

$\therefore PM^2 = 3a^2$

Taking Sq root PM = $\sqrt{3}$ a (1 M)

Δ PQR is an equilateral / triangle

$\therefore \angle R = 60^\circ$

$\angle T = 90^\circ$

$\therefore$ **Δ** PTR is $30^\circ - 60^\circ - 90^\circ$

$\therefore PT = \frac{\sqrt{3}}{2} \times PR$ (Side opp to 60°)

$= \frac{\sqrt{3}a}{2}$

$TR = \frac{1}{2} \times PR$ (Side opp to 30°)

$= \frac{1 \times a}{2}$

$= \frac{a}{2}$ (1 M)

TN = TR + RN = $\frac{a}{2}$ + a = $3^{a}/_{2}$

In Δ PTN m$\angle$ T = 90°

$\therefore$ $PN^2 = PT^2 + TN^2$ (Pythagoras theorem) (1 M)

$\therefore$ $PN^2 = \left(\frac{\sqrt{3}a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2}\right)^2$

$\therefore$ $PN^2 = \frac{3\,a^2}{4} + \frac{9a^2}{4}$

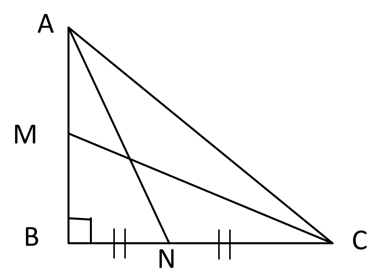
$PN^2 = \frac{12a^2}{4}$

$PN^2 = 3a^2$

Taking Sq root

PN = $\sqrt{3}$ a (1 M)

(2) In Δ ABN m$\angle$ B = 90°



(1) $AN^2 = AB^2 + BN^2$ (Pythagoras theorem)

(2) $\therefore$ $AN^2 = AB^2 + \left(\frac{1}{2}\,BC\right)^2$ (N is mid point of BC)

(3) $AN^2 = AB^2 + \frac{1}{2}\,BC^2$

Multiply both side by 4

(4) $4\,AN^2 = 4AB^2 + BC^2$ (2 M)

In Δ MBC m$\angle$ B = 90°

(5) $CM^2 = BM^2 + BC^2$ (Pythagoras theorem)

(6) $CM^2 = \left(\frac{1}{2}\mathrm{AB}\right)^2 + BC^2$ (M is mid point of AB)

(7) $CM^2 = \frac{1}{4}\,AB^2 + BC^2$

Multiply both Side by 4

(8) $4CM^2 = AB^2 + 4BC^2$ (2 M)

Adding 4 and 8

(9) $4\ AN^2 + 4\ CM^2 = 5\ AB^2 + 5\ BC^2$

(10) $4\ (AN^2 + CM^2) = 5\ (AB^2 + BC^2)$

(11) $4\ (AN^2 + CM^2) = 5\ AC^2$ (1 M)

Hence Proved